Episode 8

"The Sea of Dirac"

Meta

Dramatis personae

- Ben Tippett
- Ryan Haupt
- Jocelyn Read
- Fiona Burnell

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Timeline

00:00 - Intro

Ben: Over the course of my studies in theoretical physics, I've travelled across the continent and around the world, sampling new ideas, and tasting different answers to the questions of how, and of why. And still, I find there remains a deep hunger which lives within me - a burning desire to share these great ideas with the people around me. And so, I have assembled a team of some of the greatest, most lucid, most creative minds I've encountered in my travels, and I call them: My Titanium Physicists.

You're listening to the Titanium Physicists Podcast, and I'm Ben Tippett.

And now... allez physique!

01:12 - Theme tune

[Intro song; Tell Balgeary, Balgury Is Dead by Ted Leo and the Pharmacists]

01:49 - An introduction to Dirac's Sea

Ben: Let's preface this in really basic terms. If we take two gamma rays and smack them into each other, sometimes, sometimes matter appears. Sometimes you end up with a pair of particles - it's always a pair; a particle and its antiparticle - so say, an electron and a positron, and they fly off. Maybe you don't understand how weird this is - so, two photons go in a box, and an electron and a positron come out of the box. That's like saying if you take two bananas and you put them in a box, and then you open the box and there's a black and a white puppy inside. Because photons and particles couldn't be more different. One has mass, the other doesn't. One has charge, the other doesn't. So where does all this stuff come from? The answer's kinda neat. It turns out that historically we've known the answer to this questions before we even saw it happen in a lab.
In 1928, Paul Dirac, one of the greatest physicists of all time, came up with an equation. The key equation for quantum mechanics at the time was the Schrodinger equation. Well, Dirac wanted to generalise the Schrodinger equation so that it agreed with Einstein's theory of special relativity. The result is the Dirac equation, which allows particles not just to evolve as wave functions appropriately but it also gives them an attribute called spin. Now, the Dirac equation is incredibly successful - still used today - but it predicted some pretty gonzo stuff. For example, it predicted you could have electrons with negative energy. This posed a big problem. See, in Schrodinger quantum mechanics, the energy levels of the different quantum states an electron orbiting a nucleus could have are arranged like rungs in a ladder. The electron can metaphorically jump from rung to rung, increasing or decreasing its energy level by absorbing or emitting photons. The ladder of energy levels has no top, but it certainly has a bottom: the vacuum state. When a system is in the vacuum state, no additional energy can be extracted from it. Now, as I said before, Dirac's quantum mechanics allows negative energy levels. In other words, the ladder of energy levels now has no bottom - it goes up for an infinite number of rungs, just like in Schrodinger quantum mechanics, but now it also goes down an infinite number of rungs. This is a problem because you can always make a negative energy electron emit a photon and become more negative. Now Dirac's solution to this problem is a clever one. He came up with a picture of a universe which plugged the hole in his theory using the Pauli exclusion principle. Now, the Pauli exclusion principle says that no two electrons with the same state can be in the same place at the same time. This is why when you add electrons around a nucleus, they sort of layer themselves in quantum mechanical shells. In other words, you can't put more than

one electron on each rung of the ladder. So Dirac's answer to the negative energy question was simple. He said, "Okay - what if this ladder has no bottom, but what if each of these negative energy rungs on the ladder already has an electron on it?" This picture is counter intuitive, but it elegantly answers the particle creation problem that I mentioned at the start. The positron and the electron don't come from nowhere. Rather these colliding photons kick a negative energy electron into a positive energy state. We then recognise the hole that it leaves behind as the positron. Now, we call this description of an infinitely deep ladder of negative quantum states, filled with an infinite number of electrons, the Dirac Sea. And today, we're sailing aboard the Dirac Sea with my old old Friend Ryan Haupt. Hi Ryan.

05:17 - Today's guests

Ryan: Aw, hi Ben. I thought the Dirac Sea was like, some sort of space thing - like past Pluto. So I'm already lost.

Ben: Awesome! Ryan Haupt is the creator of the famous science podcast, "Science... Sort of" - and Ryan also writes a weekly column for "iFanboy." Okay Ryan, today I've got two of my most wonderful Titanium Physicists. Arise Dr. Jocelyn Reid.

Jocelyn: Raaaaargh!

Ben: Dr. Jocelyn did her undergraduate at UBC, her PhD at the university of Wisconsin, Milwaukee, and she's currently at the University of Mississippi working on neutron stars. Now arise Dr. Fiona Burnell.

Fiona: Boink!

Ben: Alright. Dr. Fiona did her undergraduate at UBC and her PhD at Princeton, and she's currently a condensed matter theorist at Oxford University.

Jocelyn: So. So. Okay. So, first off, let's talk about having a bunch of particles in a box. So, we use statistical mechanics to talk about a bunch of particles in a box. If they're you know, atoms of Nitrogen and such we have a gas, like air, and even though we don't know where each individual atom is travelling as it goes through this box, we can say stuff about the behaviour of all the atoms together bouncing around - like, their average energy gives you a temperature.

Ryan: Right. It's like how with diffusion we don't measure each individual particle but we know the overall behaviour of that set of particles, even without knowing...

Jocelyn: So you talk about speed of movement of particles on average and so on. [Okay.] So now you add quantum mechanics to this picture and things get a little weird. So for example...

Ryan: That seems to be a theme with quantum mechanics.

Jocelyn: Yeah. Yes. So for example, if instead of atoms that are, you know, sort of big macroscopic things bouncing into each other, you just put a bunch of electrons into the box, one of the weird things is that you can cool this down to zero temperature where it has no extra energy to give off to its surroundings, but the electrons are still moving around in excited states so they still have a bunch of energy.

Ryan: So is that just like a net energy of zero, or what's...?

Jocelyn: So what's going on is that the electrons can't all collapse down to zero energy because of that Pauli exclusion principle we were talking about earlier.

Fiona: The analogy that Ben was using with the ladder; you can picture this sort of physically like this: Instead of a ladder you can think of a sort of series of cups, if you like, at different heights, and you have a bunch of balls. And each cup can hold only one ball. So electrons are sort of like balls in cups. So basically, if you have one electron in the system it will fall down into the lowest cup, and it will sit there. But then when you put the next electron in, the lowest cup is full, so the electron then has to sit in the next cup up, and so on and so forth.

Ryan: So you're just trying to explain the Pauli exclusion principle?

Fiona: Yeah. [Okay, I'm good with that.] But, you already knew what it was, but...

Jocelyn: Yeah, and then the point is that so instead of cups, what these are are, say, momentum states, so the average energy of movement of all the electrons is higher than you would expect, so you end up with the electrons filling up all these possible states up to some level. This is the Fermi Sea - the sea of electrons that fill up to a certain state and everything below there is all filled up of electrons. This is something that you use a lot when you're talking about, say, the properties of a metal.

Ryan: Oh, okay - because metal bonds have that distributed electron field, right? They'll distribute the electrons across the metallic atoms.

Jocelyn: Yes. So there's a bunch of electrons moving through the lattice, and their overall properties are set by the top level of the sea, so you have to integrate over all of them up to the Fermi level - up to the top of the sea. Okay - so this is the Fermi Sea. So we can say that the sort of development of quantum statistics was useful in the 1920's when quantum theory is just being developed and they start to make sense of the theory of metals and how metals behave at various temperatures and things like that.

09:13 - Fermions and astrophysics

Fiona: Right. Well, it's actually, it's not just metals. It's actually any material where the interactions between electrons are not strong, is described in this way. So this isn't all materials but you know you can also make an insulator. So, the way you make an insulator is just that there's sort of some number of states available, and then there's a big jump in energy to the next accessible state. And so basically all of the states that are kind of accessible without paying a big energy cost are full, and so then the electrons can't move at all. And in a metal it's kind of the opposite thing, like there's some number of states full but there are still other states that are very close by, energetically speaking that the electrons can move into. So like, this kind of picture is the basis for our understanding of semiconductors for example, or many kinds of insulators, as well as metals.

Jocelyn: We can also - coming out from this - you actually, you can get some cool astro-physics, because this whole idea that electrons cannot all be compressed down to the lowest energy is what supports objects like White Dwarfs. So if our sun dies it's going to collapse down to a White Dwarf. And what's going to happen is that it runs out of fuel - it stops producing all the thermal energy; the heat that supports it now - and it starts cooling off and collapsing down. But it doesn't just keep on collapsing forever, like gravity keeps pulling it in and pulling it in, but what happens is all the electrons in the Sun will be confined to a certain volume, at which point they'll be basically kept from collapsing further by the Pauli exclusion principle. If you consider the entire mass of the Sun, and this is a White Dwarf. What supports the White Dwarf from collapse, is the fact that on average the electrons in that White Dwarf have to be moving at a certain speed, just because there's not enough states for all of them otherwise. And that speed gives you the pressure that supports a White Dwarf from collapsing.

Ben: So White Dwarfs are great big Fermi Seas of particles. [Yes.]

Ryan: Okay, so how do we get from a Fermi Sea to a Dirac Sea.

Ben: So a Fermi sea has a bottom...

Ryan: The bottom being the lowest level? [Yeah.]

11:16 - Relativity

Jocelyn: What Dirac was doing is saying now I want to get quantum mechanics to play nice with relativity. So I need to find an equation that going to describe these quantum particles that's compatible with special relativity, where you can shift the reference frame that you're describing the system in and the equations still have to give the same behaviour. Because in special relativity, where it doesn't matter how fast you're moving, you still have the same physics in your little lab on your spaceship. So you need a system of equations that can be translated from scientists sitting still and scientists zooming past at, you know, 95% of the speed of light.

Fiona: Now it's maybe worth mentioning that the Dirac equation is not the only thing that you can write down that does this. For example, you can describe photons relativistically - so light, relativistically - without needing to reference this idea of the Dirac Sea. So basically this comes down to the Fermi statistics versus Boson statistics. So what that means is that there are certain kinds of particles such as electrons and protons and you know, the things that make up nuclei, where this criterion applies. But not everything - there are other kinds of particles that aren't...

Ben: So the distinction is that if a particle has a half integer spin - I mentioned this word spin before, it's an attribute of a fundamental particle - if it's got a half integer spin it's called a Fermion and it obeys Fermi statistics. If you take a box full of these Fermions and you extract all the energy that you can out of it - you know, put it in a freezer at absolute zero so that it's as cold as it can get, there will still be particles moving around. To contrast, the other type of matter, if it's got an integer spin: so this counts as photons, ...

Fiona: The Higgs Boson, the infamous 'God particle' is a spin zero object.

Ben: So these Bosons don't have to obey the Pauli exclusion principle, by their nature. And so if you pull all the energy out of one of these boxes full of Bosons, each of these Boson particles will go down to the lowest energy level, and you'll just get a pile of particles that are all at same energy level. So that's the distinction between the two.

Ryan: Wait, I don't follow that. So, I thought the Pauli exclusion principle only applied to electrons in orbitals around atomic nuclei.

Jocelyn: It applies to any Fermions.

Ryan: Any Fermions.

Fiona: It applies to an electron irrespective of whether or not it's in the atom. [Yeah.] [Okay.]

Ben: So the nuclei of the atoms in most states are Fermions. I think you can… what people can do is they can take nuclei and match up the spins of the different components so that you get a Boson effectively. So you can make atomic Boson gases. Okay. So not everything is a Fermion.

13:53

Ryan: I think I understand what a Fermion is. I'm not sure how it applies to Fermi's paradox, but I'm ready to move forward.

Jocelyn: Okay, so you're Dirac and it's before 1933. You're about to win the Nobel prize in 1933 but you don't know it yet. And the first theory of relativistic quantum particles that you've come up with is to make an equation…

Ryan: Wait? Relativistic quantum particles? I thought quantum particles by their nature were non-relativistic.

Jocelyn: [No.] So. You're mixing up special relativity [Yes, I am.] and general relativity.

Ryan: Okay, I am mixed up.

Jocelyn: And quantum's... Quantum mechanics has been reconciled to play well with special relativity.

Ryan: Oh, that's good.

Ben: So, starting with the Dirac equation, quantum mechanics has played well with special relativity. General relativity is Einstein's theory of gravity, and that's the one that quantum mechanics is having trouble with.

Ryan: I'm with you now. Thank you for clearing that up.

14:44 - Predicting positrons

Ben: So let's get back on the Fermion train.

Jocelyn: Yeah, so Dirac: He's just made his equation and he looks at his equation and he goes, "Wait a sec! This predicts all these electrons with negative energy." [Was that wrong?] Well, it turns out that he turns this prediction eventually into a prediction of positrons, and that's why [Okay.] he gets the Nobel Prize in 1933, but at first, it was just like, you know, everyone's like, "Hey Dirac, your equation there, it's got a bunch of infinitely negative energy particles in it - that's a little weird." And here's where the Dirac Sea comes in. Because he's like, well, you know what? We know how Fermions behave. We're just saying it's all full up to the zero level, and all these electrons we see are just the free-floating ones above the vacuum energy, and then there's the sea of electrons going up to zero, and that's perfectly fine.

Fiona: The essence of the Dirac equation, as I see it is, for example, if you think about what happens in a White Dwarf star, and you have so many electrons, right, that there has to be a very very large number of possible states you can put these electrons in. But, basically to have many many electrons you need to have states at arbitrarily high energy. Even right there to understand how you can make this star out of electrons, you have to start thinking about this kind of tower of states of Fermions that extend up to infinitely high energy. And I think the essence of the Dirac equation is that, what it tells you is that this tower of states, that you know just from thinking about something rather concrete like a star and how you get this sort of star to not collapse - this infinitely high tower of positive energy states has - each positive energy state has a partner at negative energy, okay. That's the crisis. So having a very high energy positive energy state isn't such a problem because you just say, well, we don't have enough energy to put anything up there, so there's just nothing there. But having arbitrarily low energy states is a problem because everything's going to fall into them. And so the solution to that, that Dirac came up with is to say that, well, "Why can't anything fall into these states?" Well, the answer is because these particles are Fermions, and those states are filled. And so once they're filled nobody else can fall in.

Jocelyn: It's electrons all the way down.

Ben: Yeah.

16:55

Fiona: That's actually a good juncture to mention the materials aspect.

Ryan: Yeah - tell me more about materials.

Fiona: One of the interesting things about this idea of the Dirac Sea is… So, when you think about this in the context of thinking about relativistic quantum mechanics; you have this problem of states that go down to arbitrarily negative energy. But actually, if you think about electrons in most materials, like a metal or many insulators, you have a similar thing, which is that, as we said before, basically in the material there's a certain number of states that are accessible to the electron. So in this case the analogy of the Dirac Sea is simply the Fermi Sea, because there's only a finite number of states, and basically the material - you know, electrons it has - it will fill up that number of states, and will fill up - if it has N electrons is will fill up the N lowest energy states. And so... In that case you can ask, "What is a positron?" If I'm just talking about say a metal, you can make a sort of electron-positron pair by just taking an electron from whatever state it was sitting in - because it's in some state. And there are different electrons at different negative energies - You could kick this electron out of its low energy state and put it in a high energy state. For example, you could shoot a photon into a material. So now you have an electron at a higher energy, but you also have the absence of an electron, right, where this state was - and that is what we call a hole, and it behaves exactly like a positron. So it's something that has a positive charge. It has the opposite spin to the electron that was there. And so on.

Jocelyn: There's one last cool thing to talk about. Dirac's talking about his Dirac Sea and these holes in the Dirac Sea that act like particles [Yeah, sure.] and everyone's like "Yeah, whatever, Dirac," but then they actually detected the things.

18:32 - Detecting positrons

Ben: Yeah that's right. The first...

Ryan: They detected what? What things?

Jocelyn: Positrons!

Ben: Positrons!

Ryan: Oh, yeah yeah yeah. How do we do that?

Ben: Oh it's neat. This was in 1932. So a positron has the same mass as an electron [That makes sense.] and it's got the opposite charge. So if you put it in a magnetic field, and kick it with the same amount of energy, they'll move in a circle, but an electron will move counterclockwise in the magnetic field [anti-circle!] and a positron will move clockwise. So they'll move in opposite directions. So you can see they put it in something called a cloud chamber which lets you track the motion of charged particles by making little lines of condensation. And then they put this cloud chamber in a magnetic field, and they shot some gamma rays at a lead plate, and then they showed that there were these things that looked exactly like electrons - so they had the same ratio of mass to electric charge, but they move counterclockwise instead of moving clockwise. And so they said, "Hey! It's a positron!" [Right!] And ever since then we've had antimatter.

Ryan: Cool!

Jocelyn: It sounds like we might be ready to talk a little bit about graphene.

Ben: Aright, let's talk about graphene.

Ryan: Ooo! I know what graphene is.

Ben/Jocelyn: Okay, tell us about graphene is.

Ryan: It's a flat sheet of carbon molecules, right? [Totally!] One atom thick. And it's extremely strong. You can spin it really fast and it doesn't break.

Fiona: And my favourite fact about graphene is that the Nobel prize in physics last year was awarded for the first people who were able to produce it in the lab, and they did this - there was sort of a big race, if you like, between a few different groups who were interested in making graphene experimentally, and nobody could figure out how to grow it. There are various things that were tried, and the people who were first able to produce it did this by using pencils and Scotch tape.

Ryan: Wow! That's awesome.

Fiona: Pretty funny! It was Andre Geim and (Konstantin) Novoselov, and they're currently in Manchester, although I'm not sure that that's where they were when they did this. But they're both in Manchester.

Ben: Cool. Yeah. They just took a piece of graphite, and then they put a piece of scotch tape on it, and then tore the piece of Scotch tape off it and there was a big sheet of graphene.

Jocelyn: Actually, that's not entirely true. I mean, one of the things was they stuck it down on something where they could then locate the places in that sheet where it was only one atom thick, and they found graphene in it. So it's not like they just magically got this enormous plane of one atom thick stuff from Scotch tape and graphite. It was a bit of hunting down the actual places on this stuck down bit of graphite, where...

Fiona: Yeah, so it's very interesting that you can lift a bit of it with the Scotch tape and then you put it down on a substrate. Silicon Carbide is what they originally used, and it will stick onto the substrate so you can take the tape away and look at it under a microscope. And you can visually see the thickness of the layers. I mean, the amazing thing about this is that it's very easy to do - unlike a lot of materials, right, where you need some very specialised sophisticated apparatus just to produce the samples. The quality of graphene samples that people started working on; you just need an undergraduate student and a microscope, basically - and you can make them.

21:26 - Why graphene is special

Jocelyn: The other cool thing about graphene though, is if you look at the electronic structure: is it where the valence electrons are travelling, Fiona?

Fiona: The possible states of the electrons fall into what we call bands. Okay, and we call the valence states, are the states that are occupied by electrons, okay, and the states that are empty are often called the conduction states or the conduction band.

Jocelyn: So they're divided by the Fermi Sea?

Fiona: Right! By the surface of the Fermi Sea. So a band, is just sort of, there's a whole bunch of sort of energies that, if you make a very small change in energy, but change, for example, the momentum a little bit, there'll be another state. So it's kind of like, you can sort of draw it as a line, or a surface as you change the momentum, and the top of the Fermi Sea cuts through this. So, graphene is two dimensional, so it's a surface. So the top of the Fermi Sea cuts through this surface, as it would in any two dimensional material, but what's special about graphene is that the Fermi sea actually cuts through - so there's kind of two surface, one of which is under the Fermi Sea and one of which is above the Fermi sea, but these two surfaces both touch exactly the Fermi Sea, in a couple of places. So there's basically just a couple of special points where you have electrons that have very low energy relative to the energy of the surface of this Fermi Sea. Did that make sense, or did I say it all backwards?

Ben: No! [laughter] Well, I'm not sure if I understand it exactly. Okay. So, let's piece this together. So we have a sheet of carbon atoms. So, it's not clear when we say that there's a state in this lattice, what we're referring to. Because we're used to thinking of an electron as kind of living between the two atoms in the lattice. So what do these electron states correspond to? They correspond to, like, energies that one electron in the whole lattice can have?

23:09

Fiona: That's right. So the most basic thing you might think is that you have a bunch of atoms and they're bonded together chemically, but what the chemical bond means is that, as you just said, the electron is not committed to being on one atom or the other. [Right.] But now, a material like, for example, graphene, is just a whole bunch of carbon atoms and all of the atoms are bonded together. And so that means that the electron doesn't actually have to commit to sitting on any particular atom.

Ben: So the wave function of the one electron is spread out over the entire graphene surface.

Fiona: Absolutely. And so because it's completely spread out, then the way that we identify what it's doing is by thinking about its momentum.

Ben: Right. So the honeycomb of quantum states that the electrons can have is composed of all of the different momenta that the electrons can have over this entire lattice.

Fiona: Right! And in particular, for any momentum that you would pick you'll find, in graphene, that there are two possible states at a given momentum, and that these two states have different energies. So for each momentum there will be a high energy state and a low energy state. [Okay.] And typically one of those states - if you put the Fermi sea sort of right in the middle - in other words, if you choose to have one electron per atom in the carbon then basically, of the two states accessible the low energy state is always filled and the high energy state is always empty, because the system is half full.

Ben: So what we're saying is we've got this lattice of carbon and the quantum states of all of the electrons have filled all of the possible momentum states, but they're all in the lowest energy states.

Fiona: Right. In the lower of the two choices [Right.] that each momentum had. [Sure.] And energy.

24:45

Ben: Okay. And so that's what we mean by setting the Fermi Sea to be between the two.

Fiona: Right, exactly. Yeah, that there's sort of a low energy state and a high energy state and the low energy state is full and the high energy state is empty.

Ben: So then when we excite one of these electrons from the low energy state to the high energy state, that generates a hole in the low energy state.

Fiona: Correct. [Right.] That is a description that applies to any insulator. And what's special about graphene is that actually there are some special momenta, okay, where there is a state that sits exactly at this Fermi Sea, if you like. So actually a pair of states.

Ben: I see.

Fiona: And if you look near that pair of states and you just sort of said well, "I want a description that's valid for energies that are very close to this Fermi Sea," that description would be the Dirac equation.

Ben: So the Dirac equation - even though we're talking about electrons living on a lattice - the Dirac equation successfully describes how one electron can be kicked up into one of these slightly higher energy levels, and its hole that it leaves behind among the lower quantum states [Right] and then how those two propagate. Is that it?

25:51

Fiona: That's right. Yes. As long as you're looking at energies that are not very far from the level set by this Fermi Sea.

Ben: Hey, what determines where the level of the Fermi Sea is? Temperature?

Fiona: Well, okay. So, what I actually refer to is a low temperature description. So the characteristic temperature scale at which most metals would… High temperature systems are like in the thousands of Kelvin. I mean it's quite high compared to room temperature. So, usually a good way to think about metals is by imagining that they're at zero temperature which is when the Fermi Sea is just full.

Jocelyn: So the electrons are moving around fast enough already that sort of room temperature thermal excitations are small, compared to what they're doing just from their quantum mechanics.

Fiona: Yes. Exactly! So what determines the position of this Fermi level is simply, each carbon atom donates some number of electrons. So in graphene each carbon atom will donate, sort of, one electron that's allowed to… obviously there are other electrons that are bound to each carbon nucleus, but then there will be, on average per carbon atom, there's sort of one free electron that's allowed to wander around in this Fermi sea. And so it's the number of electrons participating in the Fermi Sea; in other words the number of electrons that are not glued to the atomic nuclei which determines the level of the Fermi Sea.

Ryan: Sounds like you guys are spending a lot of time talking about one electron moving around a sheet of graphene.

Jocelyn: Oh, but it's so cool though, because it's obeying of relativistic quantum theory. And not relativistic in terms of like travelling at the speed of light, but the equation that describes it is the same equation that describes an electron moving at near light speed. So in this table top experiment with sheets of pencil stuck on a semiconductor with Scotch tape, you can test all kinds of quantum theory predictions about how relativistic quantum particles would behave.

Ben: So, the graphene works as an analogue for special relativity tests. So then we don't need a collider to shoot of super super high muons. We can just play with electrons on this graphene sheet.

Ryan: I feel like I got the Dirac Sea, then all of a sudden graphene happened, and... [laughter] I think I get what's going on.

28:01

Ben: Okay. So the moral of the story is that the Dirac equation applies to particles moving at high speeds wandering around on their own. The neat thing about graphene is that this is a particle that's stuck to an atom. And it's stuck to an atom that's stuck to other atoms, so the fact that you end up with the Dirac equation is an emergent property. It was kind of unexpected that you'd see this space-time structure in the graphene.

Ryan: Okay. Alright, yeah - I can see that.

Jocelyn: That's why graphene has got millions of publications in the last five years.

Ryan: So I guess my final question is, why do we care about the Dirac Sea and graphene? Are there applications to this knowledge or is it still too new and fresh?

Jocelyn: There's like, all sorts of, like, if you look at graphene news articles, then, you know, there's stuff like ultra-fast transistors and DNA sequencing and flexible touchscreens, like, it's just got all sorts of cool electronic properties that will let people make neat stuff with it. But it will probably take a while before any of it is really production ready, of course.

Fiona: Yeah, and from the perspective of a lot of physicists, what Ben said is accurate. So, why is this exciting? Well, here you have something that you can make on a table top. I mean, you know, very cheaply. It's relatively easy to kind of work with, it's two dimensional, it's not toxic, it's not… okay. And you can ask questions about this mathematical structure - the same kind of questions that people might be interested in asking with particle accelerators which cost, you know, billions of dollars. If you like, it gives a probe of understanding, you know, the physics of the Dirac equation, as well as its implications for the material itself, and what you can do with the material in terms of its conductivity properties which, since it's similar to a semiconductor, you can imagine that various things that we use semiconductors for, including chips and transistors and these kinds of things; there might be graphene applications there at some point in the future.

30:05 - Closing words

Ben: Cool! Alright, so let's close this up. Fiona, Jocelyn, thank you! You've pleased me, so your efforts have borne fruit and that fruit is sweet. Here is some fruit: Joselyn, you get a Kumquat.

Jocelyn: Om, yom, yom yom yom!

Ben: And Fiona, you get an anti-kumquat.

Fiona: Chomp! Chew!

Ben: Oh! She ate it without exploding. Fiona is an anti-person. Alright, so, I'd like to thank my guest, Ryan Haupt. Thank you for coming on my show.

Ryan: Yeah, hey - thanks! This was fun. This was a good time.

Ben: You did a wonderful job.

Ryan: Did I?

Ben: Yes. People are going to be writing in, telling you how great you are.

Ryan: Or saying, "When is Charlie coming back?"

Ben: They will say, "When is Charlie going back?"

Ryan: "We miss Charlie."

Ben: Okay, so if you want to contact us to tell us how great Ryan is, you can e-mail us at barn@titaniumphysics.com or you can follow us on twitter at @titaniumphysics. You can visit our website at www.titaniumphysics.com or look for us on Facebook where we have a fan page. If you have a question you'd like my Titanium Physicists to address, e-mail your questions to tiphyter@titaniumphysics.com and if you're a physicist and would like to become one of my Titanium Physicists, e-mail physics@titaniumphysics.com. We're always recruiting. The Titanium Physics podcast is a member of Brachiolope Media. If you've enjoyed the show, you might also enjoy "Science… Sort Of"...

Ryan: Woooo!

Ben: Or "The Weekly Weinersmith"...

Ryan: Also, wooo!

Ben: So please, check them out. The intro music is by Ted Leo and the Pharmacists, and the end music is by John Vanderslice. Good day my friends, and remember to keep science in your hearts.

31:47 - Outro music

[Outro song; Angela by John Vanderslice]

Bonus Material / Cut scenes

32:38 - The Hulk

Ryan: And then Bruce Banner was hit with a gamma ray and he became the hulk.

Jocelyn: Hulk! Hulk!

Ben: Don't start Jocelyn on the Hulk, Ryan.

Ryan: Does she like the Hulk?

Ben: Yes.

Jocelyn: [Singing] ♪♫ Doctor Banner, pelted by gamma rays, turned into the Hulk, ain't he unglama-rays. Wrecking the town with the power of a bull. No monster who is as lovable as ever loving Hulk! Hulk! Hulk! ♪♫

Ben: Jocelyn, you know that you've sung on every episode you've been on so far? [laughter]

Jocelyn: Really?

Ryan: I would have done a duet, had I known the lyrics.

Ben: [sigh]

33:15 - Bees

Ben: Alright, so. In quantum mechanics, for these Fermi particles, it's like you imagine a great big beehive. And each of the little cells in the honeycomb which usually hold bees, are called quantum states, right?

Ryan: Can I ask you something really quick, Ben? Do you think that the honeycomb is like bee apartments? Is that how you view a beehive?

Ben: Well, they're larva taverns. Let's just say that each of the honeycombs [laughter] is a bee apartment. [More laughter] Alright, so imagine that it's a bee hive...

[General laughter]

Ben: Dammit! Okay.

Jocelyn: Take two, alright. I'm going to have to mute myself and laugh hysterically. Just a second...

Ben: So it's like bee apartments. And each of the little cells that the bee can go into is a quantum state, so it corresponds to the bee having a certain amount of energy and moving with a certain momentum, alright? So the bees move around from cell to cell as they go about their business. Now a Fermi Sea is when we cool down the beehive, and they get… and all the bees migrate to lower and lower cells until all of the very lowest… what are they called, honeycomb cells? All of the different cells at the bottom of the hive are filled up. And so there aren't any gaps. It's all bees filling up the cells to the first… I don't know… to knee height, and then past there there's just big empty cells.

Jocelyn: You should have the bees moving to the centre to stay warm together when it's cold outside, instead of...

Ryan: Poor worn out bees.

Ben: They're moving to ground level, okay?

Jocelyn: Right, but why did the bees move down to ground level when it's cold, Ben?

Ben: The ground is warm.

Ryan: Why are you freezing your beehive?

Ben: So the Dirac Sea is like saying, "Hey, there's no ground. It's just a beehive that goes all the way down." And so the question is, what keeps the bees from going down to these lower cells? And so Dirac's metaphorical answer was that all the bee cells up to the zero energy height are full of bees already, [laughter] and so more bees can't go into them. And then, if you hit one of those low energy, negative energy bees with some energy it can move up to one of the higher cells and that leaves a little cell open. And that cell will get filled in by other bees. But every time one bee enters it - the empty cell - it will leave an empty cell behind. And so, there'll be this little hole moving around in the bottom of the beehive. And that hole will move around just like a bee does, only it will be an anti-particle bee.

35:57 - Bee'ing a good analogy

Jocelyn: It will be an anti-bee.

Ben: It will be an anti-bee.

Ryan: Ben, I'm not sure how good an analogy it is, if you can restate the entire analogy and replace the word "bee" with the actual word that we're talking about [laughter] and the analogy still... It's the same sense.

Ben: I guess the point is…

Ryan: You've completely bodged beehive dynamics. You don't know how a honeycomb works. You've created an infinite beehive. [laughter]

Ben: Okay, so the reason is that physicists think of quantum states in really physical terms, right. And so we use that analogy of, you know, talking about quantum states that are filled and not, as if they're actually beehive cells being filled with bees. When in actuality they're metaphorical cells that live in phase space.

Ryan: Wait the whole Dirac Sea is a metaphor in the first place? There's no physical...

Jocelyn: What a minute, they live in… well okay... [fade out]